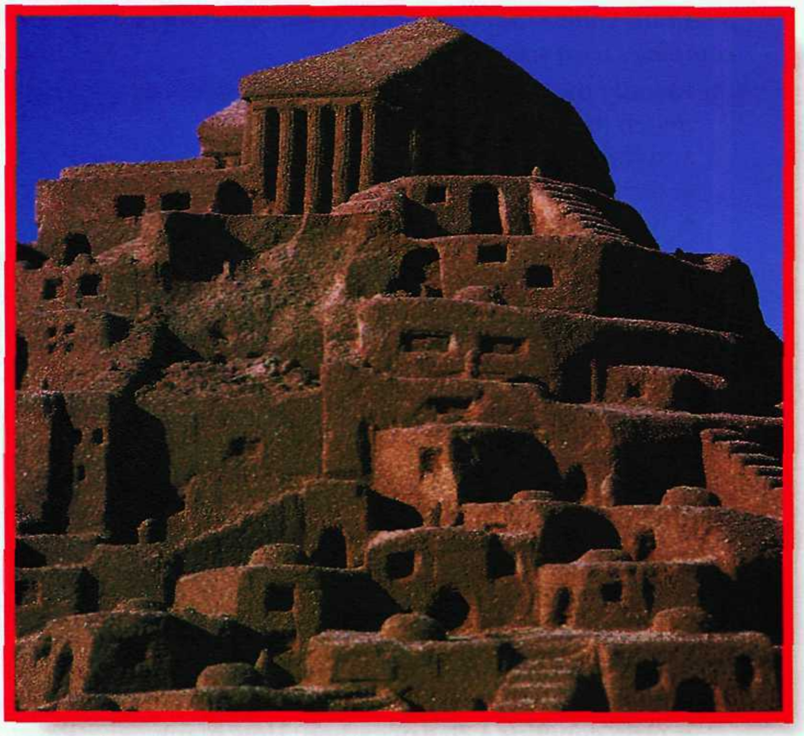
Chapter 7

CHEMICAL QUANTITIES



This sand sculpture contains millions of grains of sand.

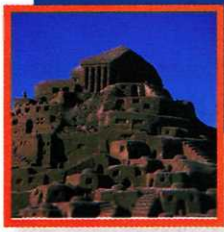
FEATURES

DISCOVER IT! COUNTING BY MEASURING MASS

You need 100 paper clips of the same size and a centigram balance.

1. Find the mass of 25 paper clips. Divide the total mass by 25 to find the average mass of a paper clip. Repeat this step using 25 different paper clips until your average masses agree.
2. Select about 75% of your paper clips and find their mass. Without counting, calculate the number of paper clips in your sample.
3. Count the number of paper clips in your sample.
4. Repeat Steps 2 and 3 with a different sample size.

Did the number of paper clips you counted in the sample (Step 3) equal the number you calculated by mass (Step 2)? Explain how you would use the balance to count out 185 paper clips. What is the advantage of using a larger sample size in Step 1? What is a disadvantage? After completing this chapter, revisit this activity and suggest other ways you might count your paper clips.



THE MOLE: A MEASUREMENT OF MATTER

objectives

- Describe how Avogadro's number is related to a mole of any substance
- Calculate the mass of a mole of any substance

key terms

- mole (mol)
- Avogadro's number
- representative particle
- gram atomic mass (gam)
- gram molecular mass (gmm)
- gram formula mass (gfm)

Every year, contestants from all over the world travel to Harrison Hot Springs in British Columbia, Canada to compete in the world championship sand sculpture contest. Each contestant creates a beautiful work of art out of millions of tiny grains of sand. **If you assume that sand is pure silicon dioxide (SiO_2), what chemical unit could you use to measure the amount of sand in a sand sculpture?**

What Is a Mole?

You live in a quantitative world. The grade you got on your last exam, the number of times you heard your favorite song on the radio yesterday, and the cost of a bicycle you would like to own are all important quantities to you. These are quantities that answer such questions as "how much?" or "how many?" Scientists spend time answering similar questions. How many kilograms of iron can be obtained from one kilogram of iron ore? How many grams of the elements hydrogen and nitrogen must be combined to make 200 grams of the fertilizer ammonia (NH_3)? These two questions illustrate that chemistry is a quantitative science. In your study of chemistry, you will analyze the composition of samples of matter. You will also perform chemical calculations relating quantities of reactants and products to chemical equations. To solve these and other problems, you will have to be able to measure the amount of matter you have.

How do you measure matter? One way is to count how many of something you have. For example, you can count the CDs in your collection or the number of pins you knock down when bowling. Another way to measure matter is to determine its mass or weight. You can buy potatoes by the kilogram or pound and gold by the gram or ounce. You can also measure matter by volume. For instance, people buy gasoline by the liter or gallon and take cough medicine by the milliliter or teaspoon. Often, more than one method of measurement—a count, a mass, a volume—can be used. For example, you can buy soda by the six-pack or by the liter. **Figure 7.2** on the following page shows how some everyday items are measured.

Figure 7.1
Jellybeans are often sold by either weight or mass.

Some of the units used when measuring always indicate a specific number of items. For example, a pair always means two. A pair of shoes is two shoes, and a pair of aces is two aces. Similarly, a dozen always means 12 (except for a baker's dozen which is 13). A dozen eggs is 12 eggs, a dozen pens is 12 pens, and a dozen donuts is 12 donuts.

Apples are commonly measured in three different ways. At a fruit stand, apples are often sold by the *count* (5 for \$2.00). In a supermarket, you usually buy apples by weight (\$0.89/pound) or *mass* (\$1.95/kg). At an orchard, you can buy apples by *volume* (\$9.00/bushel). Each of these different ways to measure apples—by count, by mass, and by volume—can be equated to a dozen apples.

By count:
1 dozen apples = 12 apples

For average-sized apples the following approximations can be used.

By mass:
1 dozen apples = 2.0 kg apples

By volume:
1 dozen apples = 0.20 bushel apples

Knowing how the count, mass, and volume of apples relate to a dozen apples allows you to convert between these units. For example, you could calculate the mass of a bushel of apples or the mass of 90 average-sized apples using conversion factors based on the unit relationships given above.

In chemistry, you will do calculations using a measuring unit called a mole. The mole, the SI unit that measures the amount of substance, is a unit just like the dozen. The mole can be related to the number of particles (a count), the mass, and the volume of an element or a compound just as a dozen was related to these three units for apples.

Figure 7.2

Items are often sold by different types of measurements, such as a count, a weight or mass, or a volume. Which of these common supermarket items are being sold by weight? By volume? By count?

Sample Problem 7-1

What is the mass of 90 average-sized apples?

1. ***ANALYZE*** ***List the knowns and the unknown.***

Knowns:
- number of apples = 90 apples
- 12 apples = 1 dozen apples
- 1 dozen apples = 2.0 kg apples

Unknown:
- mass of 90 apples = ? kg

The desired conversion is:

$$\text{number of apples} \longrightarrow \text{mass of apples}$$

This conversion can be carried out by performing the following sequence of conversions:

$$\text{number} \longrightarrow \text{dozens} \longrightarrow \text{mass of apples}$$

2. ***CALCULATE*** ***Solve for the unknown.***

The first conversion factor is $\dfrac{1 \text{ dozen apples}}{12 \text{ apples}}$.

The second conversion factor is $\dfrac{2.0 \text{ kg apples}}{1 \text{ dozen apples}}$.

Multiplying the original number of apples by these two conversion factors yields the answer in kilograms,

$$90 \cancel{\text{apples}} \times \frac{1 \cancel{\text{dozen apples}}}{12 \cancel{\text{apples}}} \times \frac{2.0 \text{ kg apples}}{1 \cancel{\text{dozen apples}}}$$

$$= 15 \text{ kg apples}$$

3. ***EVALUATE*** ***Does the result make sense?***

Because a dozen apples has a mass of 2.0 kg and 90 apples is less than 10 dozen apples, the mass should be less than 20 kg of apples (10 dozen × 2.0 kg/dozen).

Practice Problems

1. What is the mass of 0.50 bushel of apples?
2. Assume that a variety of apples has 8 seeds in each apple. How many apple seeds are in 14 kg of apples?

Chem ASAP!

Problem-Solving 1

Solve Problem 1 with the help of an interactive guided tutorial.

The Number of Particles in a Mole

In Chapter 2, you learned that matter is composed of different kinds of particles. One way to measure the amount of a substance is to count the number of particles in that substance. Because atoms, molecules, and ions are exceedingly small, the number of individual particles in a sample (even a very small sample) of any substance is very large. Counting the particles is not practical. However, you can count particles if you introduce a term that represents a specified number of particles. Just as a dozen eggs represents 12 eggs, a **mole (mol)** of a substance represents 6.02×10^{23} representative particles of that substance. The experimentally determined number 6.02×10^{23} is called **Avogadro's number,** in honor of Amedeo Avogadro di Quaregna (1776–1856).

The term **representative particle** refers to the species present in a substance: usually atoms, molecules, or formula units (ions). The representative particle of most elements is the atom. Iron is composed of iron atoms. Helium is composed of helium atoms. Seven elements, however, normally exist as diatomic molecules (H_2, N_2, O_2, F_2, Cl_2, Br_2, and I_2). The

Figure 7.3
Although Amedeo Avogadro clarified the difference between atoms and molecules, he did not calculate the value that is named after him. Avogadro's number was given his name to honor his contributions to science.

Figure 7.4
There are words other than mole used to describe a number of something—for example, a dozen (12) eggs, a gross (144) of pencils, and a ream (500 sheets) of paper.

representative particle of these elements and of all molecular compounds is the molecule. The molecular compounds water (H_2O) and sulfur dioxide (SO_2) are composed of H_2O and SO_2 molecules, respectively. For ionic compounds, the formula unit is the representative particle. The ionic compound calcium chloride is composed of $CaCl_2$ formula units. Calcium ions and chloride ions are present in a one to two ratio in this formula unit. **Table 7.1** summarizes the relationship between representative particles and moles of substances. Remember that a mole of any substance always contains 6.02×10^{23} representative particles.

Table 7.1

Representative Particles and Moles

Substance	Representative particle	Chemical formula	Representative particles in 1.00 mol
Atomic nitrogen	Atom	N	6.02×10^{23}
Nitrogen gas	Molecule	N_2	6.02×10^{23}
Water	Molecule	H_2O	6.02×10^{23}
Calcium ion	Ion	Ca^{2+}	6.02×10^{23}
Calcium fluoride	Formula unit	CaF_2	6.02×10^{23}
Sucrose	Molecule	$C_{12}H_{22}O_{11}$	6.02×10^{23}

Sample Problem 7-2

How many moles of magnesium is 1.25×10^{23} atoms of magnesium?

1. **ANALYZE** ***List the knowns and the unknown.***

   Knowns:
   - number of atoms = 1.25×10^{23} atoms Mg
   - 1 mol Mg = 6.02×10^{23} atoms Mg

   Unknown:
   - moles = ? mol Mg

   The desired conversion is:

   atoms $\longrightarrow$ moles

2. **CALCULATE** ***Solve for the unknown.***

   The conversion factor is $\dfrac{1 \text{ mol Mg}}{6.02 \times 10^{23} \text{ atoms Mg}}$.

   Multiplying atoms of Mg by the conversion factor yields the answer.

   $$1.25 \times 10^{23} \cancel{\text{atoms Mg}} \times \frac{1 \text{ mol Mg}}{6.02 \times 10^{23} \cancel{\text{atoms Mg}}}$$
   $$= 2.08 \times 10^{-1} \text{ mol Mg}$$

3. **EVALUATE** ***Does the result make sense?***

   Because the given number of atoms is less than one-fourth of Avogadro's number, the answer should be less than one-fourth mole of atoms. The answer should have three significant figures.

Practice Problems

3. How many moles is 2.80×10^{24} atoms of silicon?
4. How many molecules is 0.360 mol of water?

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Problem-Solving 4
Solve Problem 4 with the help of an interactive guided tutorial.

Now suppose you want to determine how many atoms are in a mole of a compound. To do this, you must know how many atoms are in a representative particle of the compound. This number is determined from the chemical formula. For example, each molecule of carbon dioxide (CO_2) is composed of three atoms: one carbon atom and two oxygen atoms. A mole of carbon dioxide contains Avogadro's number of CO_2 molecules. Thus a mole of carbon dioxide contains three times Avogadro's number of atoms. A molecule of carbon monoxide (CO) consists of two atoms, so a mole of carbon monoxide contains two times Avogadro's number of atoms. To find the number of atoms in a mole of a compound, you must first determine the number of atoms in a representative particle of that compound and then multiply that number by Avogadro's number. **Figure 7.5** illustrates this idea with marbles (atoms) in cups (molecules).

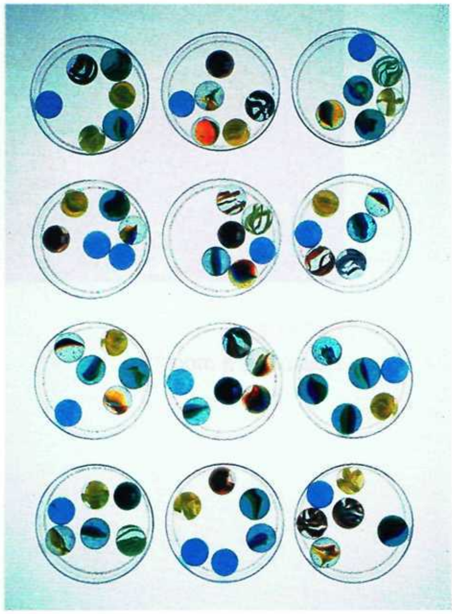

Figure 7.5
A dozen cups of marbles contain more than a dozen marbles. Similarly, a mole of molecules contains more than a mole of atoms. How many atoms are in one mole of molecules if each molecule consists of six atoms?

Sample Problem 7-3

How many atoms are in 2.12 mol of propane (C_3H_8)?

1. *ANALYZE* *List the knowns and the unknown.*

Knowns:
- number of moles = 2.12 mol C_3H_8
- 1 mol C_3H_8 = 6.02×10^{23} molecules C_3H_8
- 1 molecule C_3H_8 = 11 atoms (3 carbon atoms and 8 hydrogen atoms)

Unknown:
- number of atoms = ? atoms

The desired conversion is:

$$\text{moles} \longrightarrow \text{molecules} \longrightarrow \text{atoms}$$

Using the relationships among units given above, the desired conversion factors can be written.

2. *CALCULATE* *Solve for the unknown.*

The first conversion factor is $\dfrac{6.02 \times 10^{23} \text{ molecules } C_3H_8}{1 \text{ mol } C_3H_8}$.

The second conversion factor is $\dfrac{11 \text{ atoms}}{1 \text{ molecule } C_3H_8}$.

Multiplying the moles of C_3H_8 by the proper conversion factors yields the answer.

$$2.12 \cancel{\text{mol } C_3H_8} \times \frac{6.02 \times 10^{23} \cancel{\text{molecules } C_3H_8}}{1 \cancel{\text{mol } C_3H_8}} \times \frac{11 \text{ atoms}}{1 \cancel{\text{molecule } C_3H_8}}$$

$$= 1.4039 \times 10^{25} \text{ atoms} = 1.40 \times 10^{25} \text{ atoms}$$

3. *EVALUATE* *Does the result make sense?*

Because there are 11 atoms in each molecule of propane and more than 2 mol of propane, the answer should be more than 20 times Avogadro's number of propane molecules. The answer has three significant figures based on the three significant figures in the given measurement.

Practice Problems

5. How many atoms are there in 1.14 mol SO_3?
6. How many moles are there in 4.65×10^{24} molecules of NO_2?

Figure 7.6
How big is a mole?

Perhaps you are wondering just how large Avogadro's number is. The SI unit, the mole, is not related to the small burrowing animal of the same name shown in **Figure 7.6.** However, you can use this animal to help develop an appreciation for the size of the number 6.02×10^{23}. Assume that an average animal-mole is 15 cm long, 5 cm tall, and has a mass of 150 g. Based on this information, what is the mass of 6.02×10^{23} animal-moles?

$$6.02 \times 10^{23}\ \cancel{\text{animal-mole}} \times \frac{150\text{ g}}{1\ \cancel{\text{animal mole}}} = 9.03 \times 10^{25}\text{g}$$
$$= 9.03 \times 10^{22}\text{ kg}$$

The mass of animal-moles is equivalent to
- more than 1% of Earth's mass.
- more than 1.3 times the mass of the moon.
- more than 60 times the combined mass of Earth's oceans.

If spread over the entire surface of Earth, Avogadro's number of animal-moles would form a layer more than 8 million animal-moles thick.

What about the length of 6.02×10^{23} animal-moles?

$$6.02 \times 10^{23}\ \cancel{\text{animal-mole}} \times \frac{15\text{ cm}}{1\ \cancel{\text{animal mole}}} = 9.03 \times 10^{24}\text{cm}$$
$$= 9.03 \times 10^{19}\text{ km}$$

If lined up end-to-end, 6.02×10^{23} animal-moles would stretch from Earth to the nearest star, Alpha Centauri, more than two million times.

Suppose you could convince Avogadro's number of animal-moles to line up in 6 billion equal columns. Further suppose that each of the approximately 6 billion people on Earth counted the animal-moles in one column at the rate of 1000 animal-moles per second. Even with that many people counting that fast, it would still take more than 3000 years to count 6.02×10^{23} animal-moles! Are you beginning to understand how enormous Avogadro's number is?

The Mass of a Mole of an Element

You are always working with large numbers of atoms even if you are using microgram quantities. Even one billion atoms would be a very small amount of a substance. Working with grams of atoms is much easier. The **gram atomic mass (gam)** is the atomic mass of an element expressed in grams. For carbon, the gram atomic mass is 12.0 g. For atomic hydrogen, the gram atomic mass is 1.0 g. **Figure 7.7** shows one gram atomic mass of carbon, sulfur, mercury, and iron. Compare the gram atomic masses in the figure to the atomic masses in your periodic table. Notice that the gram atomic masses were rounded off to one place after the decimal point. All the examples and problems in this text use gram atomic masses that are rounded off in the same way. If your teacher uses a different rounding rule for gram atomic masses, your answers to problems may differ slightly from the answers given in the text.

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Animation 6
Find out how Avogadro's number is based on the relationship between the amu and the gram.

You learned previously that the atomic mass of an element (the mass of a single atom) is expressed in atomic mass units (amu). Remember that atomic masses of atoms are relative values. The atomic masses of elements found in the periodic table are weighted average masses of the isotopes of

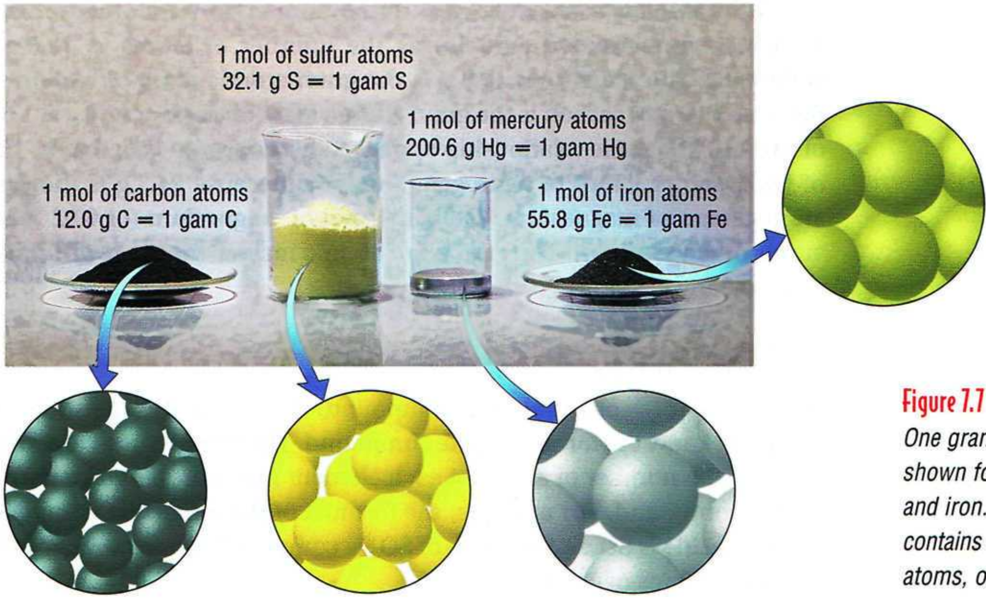


Figure 7.7
One gram atomic mass (gam) is shown for carbon, sulfur, mercury, and iron. Each of these quantities contains one mole, or 6.02×10^{23} atoms, of that substance.

that element. As you can see in **Figure 7.8,** an average carbon atom (C) with an atomic mass of 12.0 amu is 12 times heavier than an average hydrogen atom (H) with an atomic mass of 1.0 amu. Therefore 100 carbon atoms are 12 times heavier than 100 hydrogen atoms. In fact, any number of carbon atoms is 12 times heavier than the same number of hydrogen atoms, as **Figure 7.8** demonstrates. Therefore 12.0 g of carbon atoms and 1.0 g of hydrogen atoms contain the same number of atoms.

The gram atomic masses of any two elements must contain the same number of atoms. If you were to compare 12.0 g of carbon atoms with 16.0 g of oxygen atoms, you would find they contain the same number of atoms. How many atoms are contained in the gram atomic mass of an element? You are now familiar with this quantity—the gram atomic mass of any element contains 1 mol of atoms (6.02×10^{23} atoms) of that element.

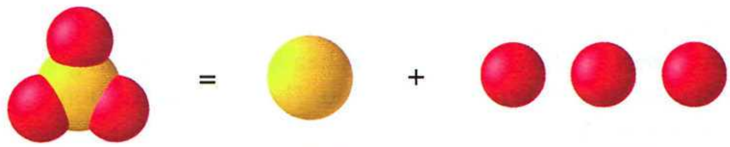
Figure 7.8
The mass ratio of equal numbers of carbon atoms to hydrogen atoms is always 12 to 1.

CARBON ATOMS		HYDROGEN ATOMS		MASS RATIO
Number	Mass (amu)	Number	Mass (amu)	$\frac{\text{Mass carbon}}{\text{Mass hydrogen}}$
	12		1	$\frac{12\text{ amu}}{1\text{ amu}} = \frac{12}{1}$
	24 [2×12]		2 [2×1]	$\frac{24\text{ amu}}{2\text{ amu}} = \frac{12}{1}$
	120 [10×12]		10 [10×1]	$\frac{120\text{ amu}}{10\text{ amu}} = \frac{12}{1}$
	600 [50×12]		50 [50×1]	$\frac{600\text{ amu}}{50\text{ amu}} = \frac{12}{1}$
Avogadro's number	$(6.02 \times 10^{23}) \times (12)$	Avogadro's number	$(6.02 \times 10^{23}) \times (1)$	$\frac{(6.02 \times 10^{23}) \times (12)}{(6.02 \times 10^{23}) \times (1)} = \frac{12}{1}$

The mole can now be defined as the amount of substance that contains as many representative particles as the number of atoms in 12.0 g of carbon-12. You know that 12.0 g is the gram atomic mass of carbon-12. Because 12.0 g of carbon is the gram atomic mass of carbon, 12.0 g is 1 mol of carbon. The same relationship applies to hydrogen as well; that is, 1.0 g of hydrogen is 1 mol of hydrogen. Similarly, because 24.3 g is the gram atomic mass of magnesium, 24.3 g is 1 mol of magnesium (or 6.02×10^{23} atoms of magnesium). Thus the gram atomic mass is the mass of 1 mol of atoms of any element.

The Mass of a Mole of a Compound

What is the mass of a mole of a compound? To answer this question, you must first know the formula of the compound. The formula of a compound tells you the number of atoms of each element in a representative particle of that compound. For example, the formula of the molecular compound sulfur trioxide is SO_3. A molecule of SO_3 is composed of one atom of sulfur and three atoms of oxygen.



SO_3 = 1 S atom + 3 O atoms

You can calculate the mass of a molecule of SO_3 by adding the atomic masses of the atoms making up the molecule. From the periodic table, the atomic mass of sulfur (S) is 32.1 amu. The mass of three atoms of oxygen is three times the atomic mass of a single oxygen atom (O): 3×16.0 amu = 48.0 amu. Thus the molecular mass of SO_3 is 32.1 amu + 48.0 amu = 80.1 amu.

1 S atom + 3 O atoms

32.1 amu + 16.0 amu + 16.0 amu + 16.0 amu = 80.1 amu

If you now substitute the unit grams for atomic mass units, you will have the gram molecular mass of SO_3. The **gram molecular mass (gmm)** of any molecular compound is the mass of 1 mol of that compound. The gmm equals the molecular mass expressed in grams. Thus 1 mol of SO_3 has a mass of 80.1 g.

Gram molecular masses may be calculated directly from gram atomic masses. For each element in a compound, find the number of grams of that element per mole of the compound. Then sum the masses of the elements in the compound. The gram molecular masses of the molecular compounds in **Figure 7.9** were obtained in this way. Try calculating the gram molecular mass values shown in **Figure 7.9** yourself. Do you get the same values? If not, review the calculation for sulfur trioxide shown above, then try again. Calculating gram molecular mass values is an important skill that you will use often in your study of chemistry.

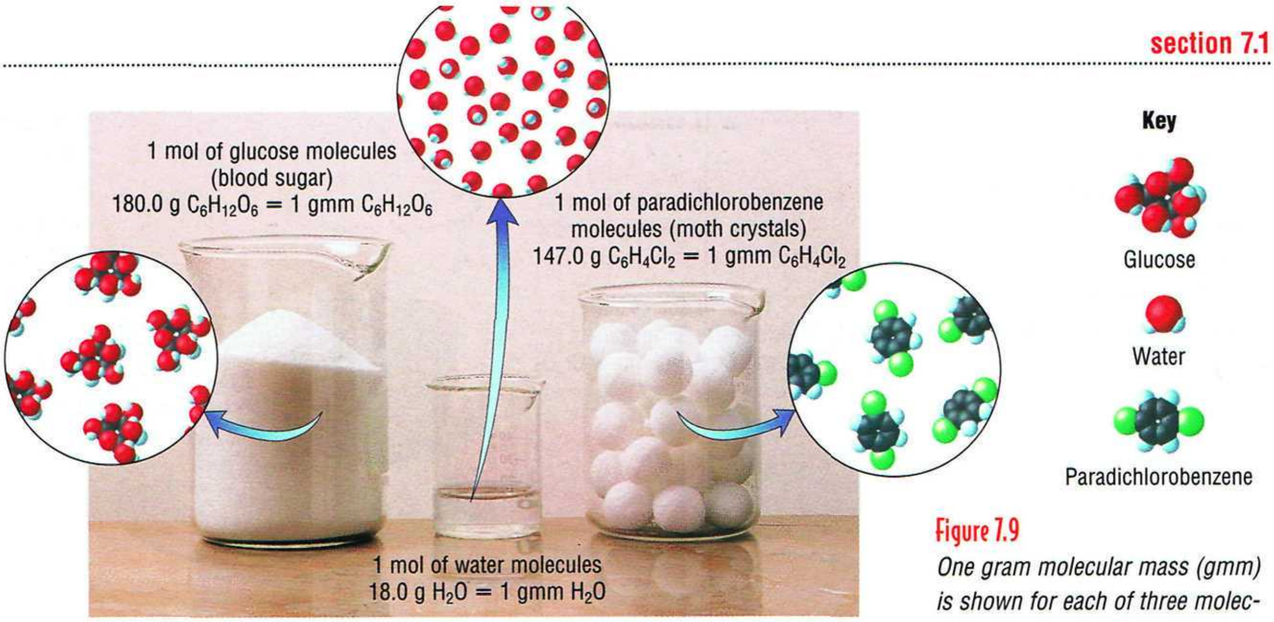


Figure 7.9
One gram molecular mass (gmm) is shown for each of three molecular compounds. Each of these quantities contains 6.02×10^{23} molecules. Do they each contain the same number of atoms?

Sample Problem 7-4

The molecular formula of hydrogen peroxide is H_2O_2. What is its gram molecular mass?

1. **ANALYZE** ***List the knowns and the unknown.***

   Knowns:
   - *molecular formula* = H_2O_2
   - 1 gam H = 1 mol H = 1.0 g H
   - 1 gam O = 1 mol O = 16.0 g O

   Unknown:
   - *gmm* = ? g

   The molecular formula gives the number of moles of each element in 1 mol of hydrogen peroxide: 2 mol of hydrogen atoms and 2 mol of oxygen atoms. Moles of atoms are converted to grams by using conversion factors (g/mol) based on the gram atomic mass of each element. The sum of the masses of the elements gives the gram molecular mass.

2. **CALCULATE** ***Solve for the unknown.***
   Use the proper conversion factors to convert moles of hydrogen and oxygen to grams of hydrogen and oxygen. Adding the results gives the answer.

$$2\,\cancel{\text{mol H}} \times \frac{1.0\text{ g H}}{1\,\cancel{\text{mol H}}} = 2.0\text{ g H}$$

$$2\,\cancel{\text{mol O}} \times \frac{16.0\text{ g O}}{1\,\cancel{\text{mol O}}} = 32.0\text{ g O}$$

$$\text{gram molecular mass of } H_2O_2 = 34.0\text{ g}$$

3. **EVALUATE** ***Does the result make sense?***
   The answer reflects the number of moles of atoms of each element and the gram atomic mass of each element. The answer is expressed to the tenth's place because the numbers being added are expressed to the tenth's place.

Practice Problems

7. Find the gram molecular mass of each compound.
   a. C_2H_6
   b. PCl_3
   c. C_3H_7OH
   d. N_2O_5
8. What is the mass of 1.00 mol of each substance?
   a. chlorine
   b. nitrogen dioxide
   c. carbon tetrabromide
   d. silicon dioxide

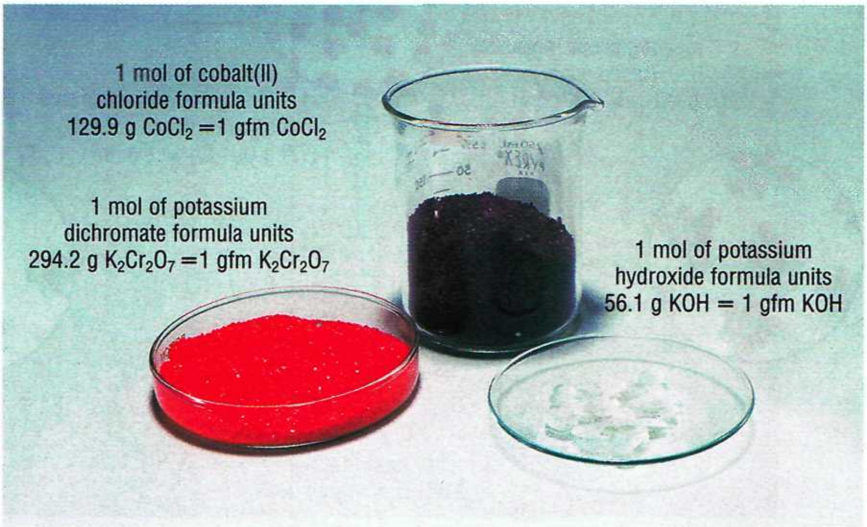


Figure 7.10
One gram formula mass (gfm) is shown for each of three ionic compounds. Each of these quantities contains 6.02×10^{23} formula units. Which of these compounds contains the greatest number of atoms?

It is inappropriate to calculate the gram molecular mass of calcium iodide (CaI_2) because it is an ionic compound. The representative particle of an ionic compound is a formula unit, not a molecule. The mass of one mole of an ionic compound is the **gram formula mass (gfm).** The gfm equals the formula mass expressed in grams. A gram formula mass is calculated the same way as a gram molecular mass: Simply sum the atomic masses of the ions in the formula of the compound. For example, the gram formula mass of calcium iodide is the gram atomic mass of calcium plus two times the gram atomic mass of iodine.

$$40.1 \text{ g Ca} + (2 \times 126.9 \text{ g I}) = 293.9 \text{ g CaI}_2$$

There are 293.9 g CaI_2 in 1 gfm or 1 mol CaI_2.

Figure 7.10 shows one gram formula mass of three ionic compounds. How many formula units are in each sample in the figure?

Sample Problem 7-5

What is the gram formula mass of ammonium carbonate ($(NH_4)_2CO_3$)?

1. ANALYZE ***List the knowns and the unknown.***

Knowns:
- formula unit = $(NH_4)_2CO_3$
- 1 gam N = 1 mol N = 14.0 g N
- 1 gam H = 1 mol H = 1.0 g H
- 1 gam C = 1 mol C = 12.0 g C
- 1 gam O = 1 mol O = 16.0 g O

Unknown:
- gfm = ? g

The formula shows that a mole of this ionic compound is composed of 2 mol of nitrogen atoms, 8 mol of hydrogen atoms, 1 mol of carbon atoms, and 3 mol of oxygen atoms. Moles of atoms are converted to grams by using conversion factors based on the gram atomic masses. The sum of the masses of the elements gives the gram formula mass.

Sample Problem 7-5 (cont.)

2. ***CALCULATE*** *Solve for the unknown.*

Using the proper conversion factors and adding the results gives the answer.

$$2\,\cancel{\text{mol N}} \times \frac{14.0\text{ g N}}{1\,\cancel{\text{mol N}}} = 28.0\text{ g N}$$

$$8\,\cancel{\text{mol H}} \times \frac{1.0\text{ g H}}{1\,\cancel{\text{mol H}}} = 8.0\text{ g H}$$

$$1\,\cancel{\text{mol C}} \times \frac{12.0\text{ g C}}{1\,\cancel{\text{mol C}}} = 12.0\text{ g C}$$

$$3\,\cancel{\text{mol O}} \times \frac{16.0\text{ g O}}{1\,\cancel{\text{mol O}}} = 48.0\text{ g O}$$

gram formula mass of $(NH_4)_2CO_3$ = 96.0 g

3. ***EVALUATE*** *Does the result make sense?*

The answer reflects the number of moles of atoms of each element and the gram atomic mass of each element. The answer is expressed to the tenth's place because the numbers being added are expressed to the tenth's place.

Practice Problems

9. Calculate the gram formula mass of each ionic compound.
   a. K_2O
   b. $CaSO_4$
   c. CuI_2
10. Find the gram formula mass of each compound.
    a. barium fluoride
    b. strontium cyanide
    c. sodium hydrogen carbonate
    d. aluminum sulfite

section review 7.1

11. Describe the relationship between Avogadro's number and one mole of any substance.
12. Find the gram formula mass of each compound.
    a. Li_2S
    b. $FeCl_3$
    c. $Ca(OH)_2$
13. How many oxygen atoms are in a representative particle of each substance?
    a. ammonium nitrate (NH_4NO_3), a fertilizer
    b. acetylsalicylic acid ($C_8H_8O_4$), the fever-reducing compound aspirin
    c. ozone (O_3), a disinfectant
    d. *nitroglycerine ($C_3H_5(NO_3)_3$), an explosive*
14. How many moles is each of the following?
    a. 1.50×10^{23} molecules NH_3
    b. 1 billion (1×10^9) molecules O_2
    c. 6.02×10^{22} molecules Br_2
    d. 4.81×10^{24} atoms Li
15. Distinguish among gram atomic mass, gram molecular mass, and gram formula mass.

Chem ASAP! Assessment 7.1 Check your understanding of the important ideas and concepts in Section 7.1.

portfolio project

Research the history of Avogadro's number. What elements other than carbon have been used to define a mole? Write a report that summarizes your findings.

section 7.2

MOLE-MASS AND MOLE-VOLUME RELATIONSHIPS

objectives

- Use the molar mass to convert between mass and moles of a substance
- Use the mole to convert among measurements of mass, volume, and number of particles

key terms

- molar mass
- standard temperature and pressure (STP)
- molar volume

If you have ever been to a circus or a carnival, you may have seen a "Guess Your Weight" booth. The person in the booth will offer to guess your weight within a certain range or you win a prize. Is the person just guessing? Probably not. Based on tables that relate average weight to height, the person can probably come fairly close to estimating your weight by estimating your height. In a similar way, chemists use relationships between quantities of matter to solve problems. **What molar relationship do chemists use to solve problems?**

The Molar Mass of a Substance

In the previous section, you learned three new terms: gram atomic mass (gam), gram molecular mass (gmm), and gram formula mass (gfm). Each is used to represent a mole of a particular kind of substance. The gram atomic mass of an element contains a mole of atoms. The gram molecular mass of a molecular compound contains a mole of molecules. The gram formula mass of an ionic compound contains a mole of formula units. Although these three terms have different specific meanings, we can use the broader term molar mass to refer to a mole of an element, a molecular compound, or an ionic compound. The **molar mass** of any substance is the mass (in grams) of one mole of the substance.

There are situations in which the term molar mass is unclear. Consider this example: What is the molar mass of oxygen? How you answer this question depends on your interpretation of it. If you assume the oxygen in the question is molecular oxygen (O_2), then the molar mass is 32.0 g (2×16.0 g)—its gram molecular mass. If you assume that the question is asking for the mass of a mole of oxygen atoms (O), then the answer is 16.0 g—its gram atomic mass. Throughout this textbook, the term molar mass is used unless there is the potential for confusion. In that case, a more specific term will be used or the formula of the substance will be given.

In the following Sample Problems, the molar mass of an element or compound is used to convert between grams and moles of a substance.

Sample Problem 7-6

How many grams are in 9.45 mol of dinitrogen trioxide (N_2O_3)?

1. ***ANALYZE*** ***List the knowns and the unknown.***

Knowns:	*Unknown:*
• number of moles = 9.45 mol N_2O_3	• mass = ? g N_2O_3
• 1 mol N_2O_3 = 76.0 g N_2O_3	

The number of grams of the compound must be calculated from the known number of moles of the compound. The desired conversion is moles $\longrightarrow$ grams.

Sample Problem 7-6 (cont.)

2. ***CALCULATE*** *Solve for the unknown.*

Multiply the given number of moles by the proper conversion factor relating moles of N_2O_3 to grams of N_2O_3.

$$9.45 \cancel{\text{mol } N_2O_3} \times \frac{76.0 \text{ g } N_2O_3}{1.00 \cancel{\text{mol } N_2O_3}} = 718.2 \text{ g } N_2O_3$$

$$= 718 \text{ g } N_2O_3$$

3. ***EVALUATE*** *Does the result make sense?*

Because 1 mol N_2O_3 has a mass of 76.0 g, and there are almost ten moles of the compound, the answer should be about 700. The answer has been rounded to the correct number of significant figures.

Practice Problems

16. *Find the mass, in grams, of each.*
- **a.** 3.32 mol K
- **b.** 4.52×10^{-3} mol $C_{20}H_{42}$
- **c.** 0.0112 mol K_2CO_3

17. Calculate the mass, in grams, of 2.50 mol of each substance.
- **a.** sodium sulfate
- **b.** iron(II) hydroxide

Chem ASAP!

Problem-Solving 16

Solve Problem 16 with the help of an interactive guided tutorial.

Whereas Sample Problem 7-6 used a conversion factor based on the molar mass to convert moles to grams, the following Sample Problem does the reverse, using a conversion factor based upon the molar mass to convert grams to moles.

Sample Problem 7-7

Find the number of moles in 92.2 g of iron(III) oxide (Fe_2O_3).

1. ***ANALYZE*** *List the knowns and the unknown.*

Knowns:
- mass = 92.2 g Fe_2O_3
- 1 mol Fe_2O_3 = 159.6 g Fe_2O_3

Unknown:
- number of moles = ? mol Fe_2O_3

From a known number of grams of a compound, the unknown number of moles of the compound must be calculated. The desired conversion is grams $\longrightarrow$ moles.

2. ***CALCULATE*** *Solve for the unknown.*

Multiply the given mass by the proper conversion factor relating mass of Fe_2O_3 to moles of Fe_2O_3.

$$92.2 \text{ g} \cancel{Fe_2O_3} \times \frac{1.00 \text{ mol } Fe_2O_3}{159.6 \text{ g} \cancel{Fe_2O_3}} = 0.5776 \text{ mol } Fe_2O_3$$

$$= 0.578 \text{ mol } Fe_2O_3$$

3. ***EVALUATE*** *Does the result make sense?*

Because the given mass (about 90 g) is slightly larger than the mass of one-half mole of Fe_2O_3 (about 160 g), the answer should be slightly larger than one-half (0.5) of a mole.

Practice Problems

18. Find the number of moles in each quantity.
- **a.** 3.70×10^{-1} g B
- **b.** 27.4 g TiO_2
- **c.** 847 g $(NH_4)_2CO_3$

19. Calculate the number of moles in 75.0 g of each substance.
- **a.** dinitrogen trioxide
- **b.** nitrogen gas
- **c.** sodium oxide

Figure 7.11
The volume of eleven 2-liter soda bottles is 22 L. The volume of 1 mole of any gas at STP is a little more, 22.4 L.

The Volume of a Mole of Gas

If you look back at **Figure 7.9** on page 179, you will see that the volumes of one mole of different solid and liquid substances are not the same. For example, the volume of a mole of glucose (blood sugar) and one mole of paradichlorobenzene (moth crystals) are much larger than that of a mole of water. Unlike liquids and solids, the volumes of moles of gases are much more predictable under the same physical conditions.

The volume of a gas varies with a change in temperature or a change in pressure. Because of this variation, the volume of a gas is usually measured at a **standard temperature and pressure (STP).** Standard temperature is 0 °C. Standard pressure is 101.3 kPa, or 1 atmosphere (atm). At STP, 1 mol of any gas occupies a volume of 22.4 L. **Figure 7.11** should give you an idea of the volume occupied by 22.4 L. This quantity, 22.4 L, is known as the **molar volume** of a gas and is measured at STP. Because 1 mol of any substance contains Avogadro's number of particles, 22.4 L of any gas at STP contains 6.02×10^{23} representative particles of that gas. Would these values differ for gaseous elements compared with gaseous compounds?

Would 22.4 L of one gas also have the same mass as 22.4 L of another gas at STP? Probably not. A mole of a gas (22.4 L at STP) has a mass equal to its molar mass. Only gases with the same molar masses would have equal masses for equal volumes at STP.

Sample Problem 7-8

Determine the volume, in liters, of 0.60 mol SO_2 gas at STP.

1. *ANALYZE* *List the knowns and the unknown.*

Knowns:
- moles = 0.60 mol SO_2
- 1 mol SO_2 = 22.4 L SO_2

Unknown:
- volume = ? L SO_2

The known is the number of moles and the unknown is the number of liters of SO_2. Use the relationship 1.00 mol SO_2 = 22.4 L SO_2 (at STP) to write the conversion factor needed to perform the conversion of moles $\longrightarrow$ liters.

2. *CALCULATE* *Solve for the unknown.*

$$0.60\ \cancel{\text{mol } SO_2} \times \frac{22.4\ \text{L } SO_2}{1\ \cancel{\text{mol } SO_2}} = 13.44 = 13\ \text{L } SO_2$$

3. *EVALUATE* *Does the result make sense?*

Because 1 mol of any gas at STP has a volume of 22.4 L, 0.60 mol should have a volume slightly larger than 22.4/2 = 11.2 L. The answer should have two significant figures.

Practice Problems

20. What is the volume at STP of these gases?
- **a.** 3.20×10^{-3} mol CO_2
- **b.** 0.960 mol CH_4
- **c.** 3.70 mol N_2

21. Assuming STP, how many moles are in these volumes?
- **a.** 67.2 L SO_2
- **b.** 0.880 L He
- **c.** 1.00×10^3 L C_2H_6

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Problem-Solving 20
Solve Problem 20 with the help of an interactive guided tutorial.

The density of a gas is usually measured in the units grams per liter (g/L). The experimentally determined density of a gas at STP is used to calculate the molar mass of that gas. The gas can be an element or a compound. As you can see in **Figure 7.12,** whether a gas-filled balloon sinks or floats is determined by the density of the gas in the balloon compared with the density of the surrounding air.

Sample Problem 7-9

The density of a gaseous compound containing carbon and oxygen is 1.964 g/L at STP. Determine the molar mass of the compound.

1. ANALYZE ***List the knowns and the unknown.***

Knowns:
- density = 1.964 g/L
- 1 mol (gas at STP) = 22.4 L

Unknown:
- molar mass = ? g/mol

Use the relationship 1 mol (gas at STP) = 22.4 L to write the conversion factor needed to perform the required conversion.

$$\frac{\text{g}}{\text{L}} \longrightarrow \frac{\text{g}}{\text{mol}}$$

2. CALCULATE ***Solve for the unknown.***

$$\frac{1.964\ \text{g}}{1\ \cancel{\text{L}}} \times \frac{22.4\ \cancel{\text{L}}}{1\ \text{mol}} = 43.9936 = 44.0\ \text{g/mol}$$

3. EVALUATE ***Does the result make sense?***

The ratio of the calculated mass (44.0 g) to the volume (22.4 L) is about two, which is close to the known density. The answer should have three significant figures.

Practice Problems

22. A gaseous compound composed of sulfur and oxygen that is linked to the formation of acid rain has a density of 3.58 g/L at STP. What is the molar mass of this gas?

23. What is the density of krypton gas at STP?

Chem ASAP!

Problem-Solving 22

Solve Problem 22 with the help of an interactive guided tutorial.

The Mole Road Map

You have now examined a mole in terms of particles, mass, and volume of gases at STP. **Figure 7.13** on page 186 summarizes these relationships and illustrates the importance of the mole. To convert from one unit to another, you use the mole as an intermediate step. The form of the conversion factor depends on whether you are going from moles or to moles. You use the mole conversion factor in the same way you used the unit dozen to convert among mass, volume, and number of apples in Section 7.1. According to **Figure 7.13** on the following page, how many conversion factors are needed to convert from the mass of a gas to the volume of a gas (at STP)?

Figure 7.12

Helium is less dense than air. Balloons filled with helium must be tied to a heavy object to prevent them from floating away. The balloons sitting on the table are not tied down. How does the density of the gas in these balloons compare with the density of helium?

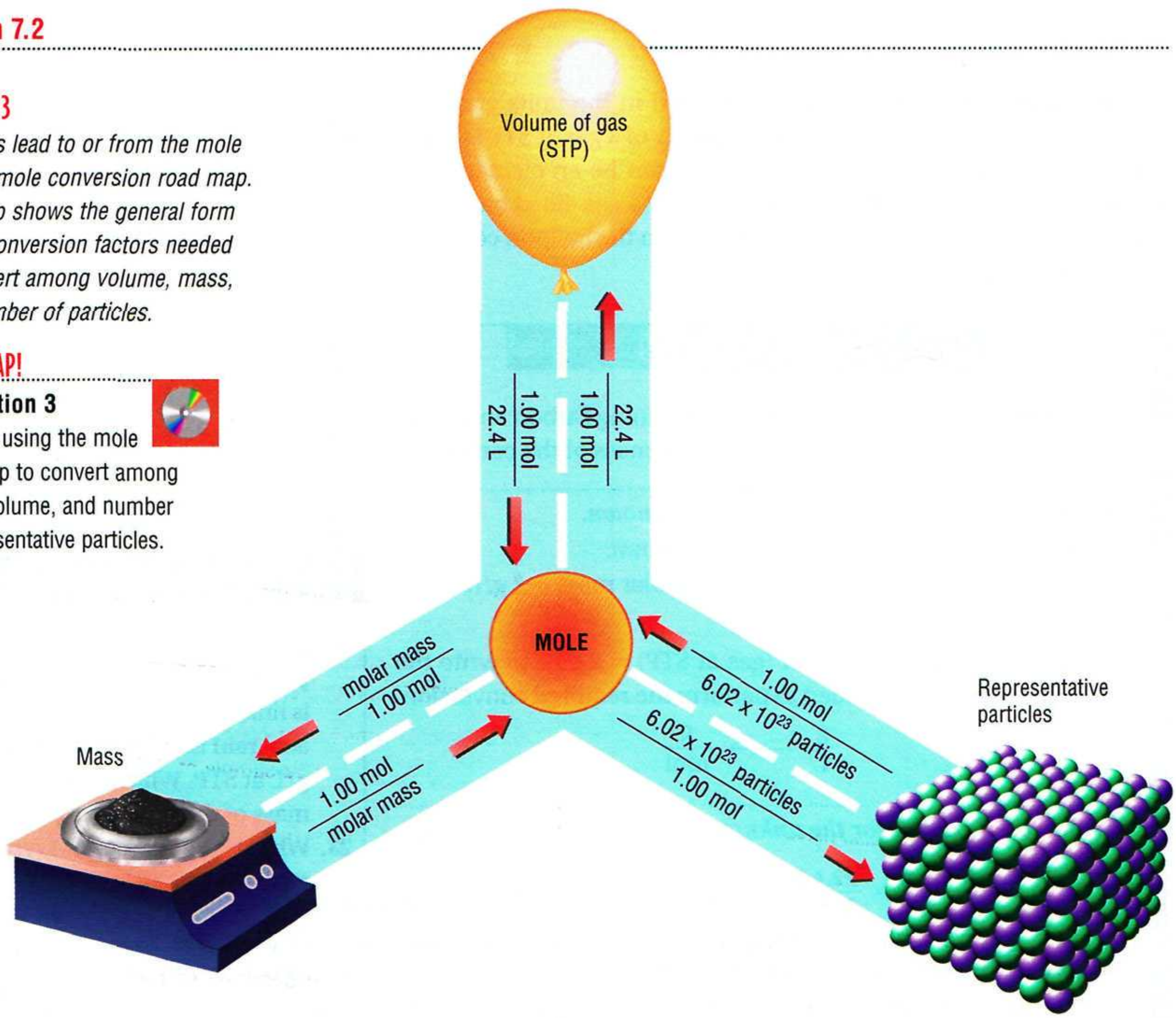


Figure 7.13
All paths lead to or from the mole on this mole conversion road map. The map shows the general form of the conversion factors needed to convert among volume, mass, and number of particles.

Chem ASAP!

Simulation 3
Practice using the mole road map to convert among mass, volume, and number of representative particles.

section review 7.2

24. Find the mass in grams of each quantity.

a. 0.720 mol Be **c.** 0.160 mol H_2O_2

b. 2.40 mol N_2 **d.** 5.08 mol $Ca(NO_3)_2$

25. **a.** Calculate the number of molecules in 60.0 g NO_2.

b. Calculate the volume, in liters, of 3.24×10^{22} molecules Cl_2 (STP).

c. Calculate the mass, in grams, of 18.0 L CH_4 (STP).

26. Would three balloons, each containing the same number of molecules of a different gas at STP, have the same mass or the same volume? Explain.

27. Find the number of moles in each quantity.

a. 5.00 g hydrogen molecules **c.** 187 g Al

b. 0.000264 g Li_2HPO_4 **d.** 333 g SnF_2

28. The densities of gases A, B, and C are 1.25 g/L, 2.86 g/L, and 0.714 g/L, respectively. Calculate the molar mass of each substance. Identify each substance as ammonia (NH_3), sulfur dioxide (SO_2), chlorine (Cl_2), nitrogen (N_2), or methane (CH_4).

Chem ASAP! **Assessment 7.2** Check your understanding of the important ideas and concepts in Section 7.2.

SMALL-SCALE LAB

MEASURING MASS AS A MEANS OF COUNTING

SAFETY

Wear your safety glasses and follow standard safety *procedures as outlined on page 18.*

PURPOSE

To determine the mass of several chemical compound samples and use the data to count atoms.

MATERIALS

- pencil
- paper
- plastic spoon
- chemicals shown in Figure A
- balance
- ruler

PROCEDURE

Measure the mass of one level teaspoon of sodium chloride (NaCl), water (H_2O), and calcium carbonate ($CaCO_3$). Make a table similar to Figure A to record your measured and calculated data.

	$H_2O(l)$	$NaCl(s)$	$CaCO_3(s)$
Mass (grams)			
Molar Mass (g/mol)			
Moles of each compound			
Moles of each element			
Atoms of each element			

Figure A

ANALYSIS

Using your data, record the answers to the following *questions in or below your data table.*

1. Calculate the moles of NaCl contained in one level teaspoon and record the result in your table.

$$\text{moles of NaCl} = ?\ \text{g NaCl} \times \frac{1\ \text{mol NaCl}}{58.5\ \text{g}}$$

2. Repeat Step 1 for the other compounds in Figure A. Use the periodic table if necessary to calculate the molar mass of water and calcium carbonate.

3. Calculate the moles of each element present in the teaspoon-sized sample of H_2O.

$$\textit{moles of } H = ?\ \textit{mol}\ H_2O \times \frac{2\ \text{mol H}}{1\ \text{mol}\ H_2O}$$

Repeat for all the other compounds in your table.

4. Calculate the number of atoms of each element present in the teaspoon-sized sample of H_2O.

$$\text{atoms of H} = ?\ \text{mol H} \times \frac{6.02 \times 10^{23}\ \text{atoms H}}{1\ \text{mol}\ H_2O}$$

Repeat for all the other compounds in your table.

5. Which of the three teaspoon-sized samples contains the greatest number of moles?

6. Which of the three compounds contains the most atoms?

YOU'RE THE CHEMIST!

The following small-scale activities allow you to develop your own procedures and analyze the results.

1. **Design It!** Can you use the technique of measuring volume as a means of counting? Design and carry out an experiment to do it!

2. **Design It!** Design an experiment that will determine the number of atoms of calcium, carbon, and oxygen it takes to write your name on the chalkboard with a piece of chalk. Assume chalk is 100 percent calcium carbonate, $CaCO_3$.

Ideal or Real?

IDEAL GASES

section 12.4

objectives

- Calculate the amount of gas at any specified conditions of pressure, volume, and temperature
- Distinguish between ideal and real gases

key terms

- ideal gas constant (R)
- ideal gas law

What if someone told you that all of the gas laws you just learned were wrong! In some ways, the person would be correct. The gas laws assume that gases behave in an ideal way, obeying the assumptions of kinetic theory. As it turns out, gases do not behave exactly this way. **What then is an ideal gas, and under what conditions do the gas laws apply?**

Ideal Gas Law

Up to this point in this textbook, you have worked with three variables regarding gas behavior: pressure, volume, and temperature. There is a fourth variable still to be considered: the amount of gas in the system, expressed in terms of the number of moles. Suppose you want to calculate the number of moles (n) of a gas in a fixed volume at a known temperature and pressure, as illustrated in **Figure 12.16.** The calculation of moles is possible by modifying the combined gas law. You can understand the modification by recognizing that the volume occupied by a gas at a specified temperature and pressure must depend on the number of gas particles. The number of moles of gas is directly proportional to the number of particles. Hence, moles must be directly proportional to volume as well. Therefore you can introduce moles into the combined gas law by dividing each side of the equation by n.

$$\frac{P_1 \times V_1}{T_1 \times n_1} = \frac{P_2 \times V_2}{T_2 \times n_2}$$

This equation shows that $(P \times V)/(T \times n)$ is a constant. This constancy holds for what are called ideal gases. A gas behaves ideally if it conforms to the gas laws. Ideal behavior depends upon certain conditions, as you will see later in this section.

If you could evaluate the constant $(P \times V)/(T \times n)$, you could then calculate the number of moles of gas at any specified values of P, V, and T. This constant is symbolized as R.

You can find the actual value of R, given an important fact about gases: 1 mol of every gas occupies 22.4 L at STP (101.3 kPa and 273 K). Inserting the values of P, V, T, and n into the equation:

$$R = \frac{P \times V}{T \times n} = \frac{101.3 \text{ kPa} \times 22.4 \text{ L}}{273 \text{ K} \times 1 \text{ mol}} = 8.31 \text{ (L} \times \text{kPa)/(K} \times \text{mol)}$$

The **ideal gas constant** *(R)* has the value 8.31 (L × kPa) / (K × mol). Rearranging the equation for R, you obtain the usual form of the **ideal gas law:**

$$R = \frac{P \times V}{T \times n}$$

$$\text{or } P \times V = n \times R \times T$$

An advantage of the ideal gas law over the combined gas law is that it permits you to solve for the number of moles of a contained gas when P, V, and T are known.

Figure 12.16
When the temperature, pressure, and volume of a gas are known, you can use the ideal gas law to calculate the number of moles of the gas.

Sample Problem 12-5

You fill a rigid steel cylinder that has a volume of 20.0 L with nitrogen gas $(N_2)(g)$ to a final pressure of 2.00×10^4 kPa at 28 °C. How many moles of $N_2(g)$ does the cylinder contain?

1. ANALYZE ***List the knowns and the unknown.***

Knowns:
- $P = 2.00 \times 10^4$ kPa
- $V = 20.0$ L
- $T = 28$ °C

Unknown:
- $n = ?$ mol $N_2(g)$

Use the known values and the ideal gas law to calculate the unknown (n).

2. CALCULATE ***Solve for the unknown.***

Convert degrees Celsius to kelvins.

$$28\ °\text{C} + 273 = 301\ \text{K}$$

Rearrange the ideal gas law to isolate n.

$$n = \frac{P \times V}{R \times T}$$

Substitute the known quantities P, V, R, and T into the equation and solve.

$$n = \frac{2.00 \times 10^4\ \cancel{\text{kPa}} \times 20.0\ \cancel{\text{L}}}{8.31 \frac{\cancel{\text{L}} \times \cancel{\text{kPa}}}{\cancel{\text{K}} \times \text{mol}} \times 301\ \cancel{\text{K}}} = 160\ \text{mol}\ N_2(g)$$

$$= 1.60 \times 10^2\ \text{mol}\ N_2(g)$$

3. EVALUATE ***Does the result make sense?***

The gas is at a high pressure, but the volume is not large. This means that a large number of moles of gas must be compressed into the volume. The large answer is thus reasonable, and the units have canceled correctly.

Practice Problems

22. When the temperature of a rigid hollow sphere containing 685 L of helium gas is held at 621 K, the pressure of the gas is 1.89×10^3 kPa. How many moles of helium does the sphere contain?

23. What pressure will be exerted by 0.450 mol of a gas at 25 °C if it is contained in a 0.650-L vessel?

Figure 12.17

Natural gas (mainly methane) is often found in underground pockets near petroleum reserves. In the past, excess gas was simply burned off, as shown here, involving a great waste of this energy resource.

Sample Problem 12-6

A deep underground cavern contains 2.24×10^6 L of methane gas $(CH_4)(g)$ at a pressure of 1.50×10^3 kPa and a temperature of 42 °C. How many kilograms of CH_4 does this natural-gas deposit contain?

1. **ANALYZE** *List the knowns and the unknown.*

Knowns:
- $P = 1.50 \times 10^3$ kPa
- $V = 2.24 \times 10^6$ L
- $T = 42$ °C

Unknown:
- $m = ?$ kg CH_4

Calculate the number of moles (n) using the ideal gas law. Convert moles to grams, using the molar mass of methane, and then convert grams to kilograms.

2. **CALCULATE** *Solve for the unknown.*

Convert degrees Celsius to kelvins.

$$42\ °\text{C} + 273 = 315\ \text{K}$$

Rearrange the equation for the ideal gas law to isolate n, the number of moles of methane.

$$n = \frac{P \times V}{R \times T}$$

Substitute the known quantities into the equation to find the number of moles of methane.

$$n = \frac{1.50 \times 10^3\ \cancel{\text{kPa}} \times (2.24 \times 10^6\ \cancel{\text{L}})}{8.31\ \frac{\cancel{\text{L}} \times \cancel{\text{kPa}}}{\cancel{\text{K}} \times \text{mol}} \times 315\ \cancel{\text{K}}} = 1.28 \times 10^6\ \text{mol } CH_4$$

Convert moles of methane to grams.

$$\text{molar mass } CH_4 = \left(4\ \cancel{\text{mol H}} \times \frac{1.0\ \text{g H}}{1\ \cancel{\text{mol H}}}\right) + \frac{12.0\ \text{g C}}{1\ \text{mol C}}$$
$$= 16.0\ \text{g } CH_4/\text{mol } CH_4$$

A mole–mass conversion gives the number of grams of methane.

$$1.28 \times 10^6\ \cancel{\text{mol } CH_4} \times \frac{16.0\ \text{g } CH_4}{1\ \cancel{\text{mol } CH_4}} = 20.5 \times 10^6\ \text{g } CH_4$$
$$= 2.05 \times 10^7\ \text{g } CH_4$$

Convert this answer to kilograms.

$$2.05 \times 10^7\ \cancel{\text{g}}\ CH_4 \times \frac{1\ \text{kg}}{1000\ \cancel{\text{g}}} = 2.05 \times 10^4\ \text{kg } CH_4$$

3. **EVALUATE** *Does the result make sense?*

The volume and pressure of the methane are very large. It is reasonable that the cavern contains the large mass of methane gas found as the solution to the problem. Also, the units canceled correctly, and the answer is expressed to the proper number of significant figures.

Practice Problems

24. A child has a lung capacity of 2.20 L. How many grams of air do her lungs hold at a pressure of 102 kPa and a normal body temperature of 37 °C? Air is a mixture, but you may assume an average molar mass of 29 g/mol for air because air is about 20% O_2 (molar mass 32) and 80% N_2 (molar mass 28).

25. What volume will 12.0 g of oxygen gas $(O_2)(g)$ occupy at 25 °C and a pressure of 52.7 kPa?

Chem ASAP!

Problem-Solving 24

Solve Problem 24 with the help of an interactive guided tutorial.

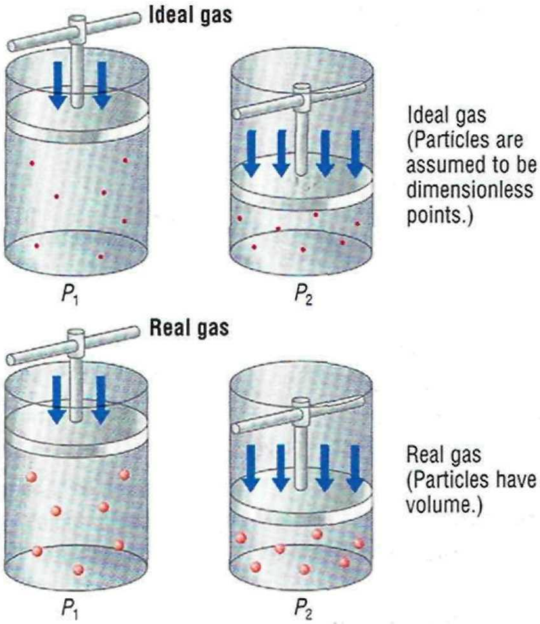


Figure 12.20
As pressure increases, the actual volumes of individual gas molecules in a real gas are no longer insignificant. It becomes increasingly difficult to compress the gas beyond a certain point, no matter what pressure is applied.

As you have read, simple kinetic theory assumes that gas particles are not attracted to each other and that the particles have no volume. These assumptions are incorrect. Gases and vapors could not be liquefied if there were no attractions between molecules. Also real gases are made up of actual physical particles, which do have a volume, as **Figure 12.20** illustrates.

The intermolecular forces that tend to hold the particles in a gas together effectively reduce the distance between particles. The gas therefore occupies less volume than is expected by the no-attractions assumption of the kinetic theory. This fact, considered alone, causes the $(P \times V)/(n \times R \times T)$ ratio to tend to be less than 1. At the same time, the molecules themselves occupy some volume, thus contradicting the zero-volume assumption of kinetic theory. This fact, on its own, causes the $(P \times V)/(n \times R \times T)$ ratio to tend to be greater than 1. One or the other of these two effects will usually dominate. In portions of the curves below the line, the intermolecular attractions dominate, causing the total volume to be less than ideal. In portions of the curves above the line, the effect of volume of the molecules dominates, causing the total volume to be greater than ideal. The temperature of the gas determines which of these two effects is the dominant one.

Compare the curves for $CH_4(g)$ at 0 °C and at 200 °C. At 0 °C, the methane molecules are moving relatively slowly. The attractions between the molecules are sufficiently strong so that, at low pressures, the curve is below the $(P \times V)/(n \times R \times T) = 1$ line. At higher pressures, the space between the molecules is reduced. The actual physical volume of the methane molecules now becomes important, and the curve is above the $(P \times V)/(n \times R \times T) = 1$ line. Raising the temperature to 200 °C increases the average kinetic energy of the methane molecules sufficiently to overcome the weak intermolecular attractive effects. Thus $(P \times V)/(n \times R \times T)$ is nearly equal to 1 at lower pressure at the elevated temperature. The ratio increases to greater than 1 only when the volume of the individual gas particles becomes important, as it does at high pressure.

LINK TO PHYSICS

Cryostats

The containers that store and transport liquefied gases are called cryostats. Their design prevents heat from being transferred from the environment to

the very cold liquid inside. The most widely used cryostats are called Dewar flasks, named after the Scottish scientist Sir James Dewar, who invented the containers in 1892. Dewar flasks are double-walled vessels with a vacuum between the walls, similar to the familiar Thermos™ bottle used to carry hot or cold drinks. Cryostats are very lightweight in comparison with compressed-gas cylinders. A given amount of a substance has a much smaller volume as a liquid than as a gas, even if the gas is under pressure. For these reasons, many gases are stored and transported in their liquefied form rather than as gases.

MINI LAB

Carbon Dioxide from Antacid Tablets

PURPOSE

To measure the amount of carbon dioxide gas given off when antacid tablets dissolve in water.

MATERIALS

- pencil
- graph paper
- tape measure
- clock or watch
- 3 rubber balloons (round)
- plastic medicine dropper
- water
- 6 antacid tablets
- pressure sensor (optional)

PROCEDURE

Sensor version available in the Probeware Lab Manual.

1. Break six antacid tablets into small pieces. Keep the pieces from each tablet in a separate pile. Put the pieces from one tablet into the first balloon. Put the pieces from two tablets into a second balloon. Put the pieces from three tablets into a third balloon.
2. After you use the medicine dropper to squirt about 5 mL of cold water into each balloon, immediately tie off each balloon.
3. Shake the balloons to mix the contents. Allow the contents to warm to room temperature.
4. Carefully measure and record the circumference of each balloon several times during the next 20 minutes.
5. Use the maximum circumference of each balloon to calculate the volume of each balloon. Assume that the balloons are spherical. (*Hint:* Volume of a sphere = $\frac{4\pi r^3}{3}$)

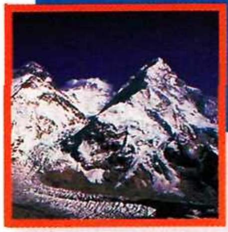
ANALYSIS AND CONCLUSIONS

1. Make a graph of balloon volume versus number of tablets. According to your graph, describe the relationship between the number of tablets used and the volume of the balloon.
2. Assume that the balloon is filled with carbon dioxide gas at 20 °C and standard pressure. Calculate the mass and the number of moles of CO_2 in each balloon at maximum inflation.
3. If a typical antacid tablet contains 2.0 g of sodium hydrogen carbonate, show how your carbon dioxide results compare with the theoretical values.

section review 12.4

26. How is it possible to calculate the amount of gas in a sample at given conditions of temperature, pressure, and volume?
27. What is the difference between an ideal gas and a real gas?
28. Explain the meaning of this statement: "No gas exhibits ideal behavior at all temperatures and pressures." At what conditions do real gases behave like ideal gases? Why?
29. Determine the volume occupied by 0.582 mol of a gas at 15 °C if the pressure is 81.8 kPa.
30. If 28.0 g of methane gas (CH_4) are introduced into an evacuated 2.00-L gas cylinder at a temperature of 35 °C, what is the pressure inside the cylinder? Note that the volume of the gas cylinder is constant.

Chem ASAP! **Assessment 12.4** Check your understanding of the important ideas and concepts in Section 12.4.



GAS MOLECULES: MIXTURES AND MOVEMENTS

objectives
- State Avogadro's hypothesis, Dalton's law, and Graham's law
- Calculate moles, masses, and volumes of gases at STP
- Calculate partial pressures and rates of effusion

key terms
- Avogadro's hypothesis
- partial pressure
- Dalton's law of partial pressures
- diffusion
- effusion
- Graham's law of effusion

The top of Mount Everest is more than 29 000 feet above sea level. In addition to tents, food, warm clothes, and ropes, an expedition to climb Mount Everest requires cylinders of oxygen. Humans need a partial pressure of oxygen of at least 10.67 kPa to survive—continual exposure to less oxygen will result in death! **What is meant by the partial pressure of oxygen, and why does it decrease as altitude increases?**

Avogadro's Hypothesis

The particles that make up different gases are not the same size. For example, chlorine molecules have large numbers of electrons, protons, and neutrons. They are bigger and occupy more volume than hydrogen molecules, which have only two protons and two electrons. Early scientists recognized that there must be such size differences and assumed that collections of larger molecules must have larger volumes than collections of an equal number of small molecules. Thus many scientists reacted in disbelief in 1811 when they heard of **Avogadro's hypothesis:** Equal volumes of gases at the same temperature and pressure contain equal numbers of particles. It was as if Avogadro were suggesting that two rooms of the same size could be filled by the same number of objects, regardless of whether the objects were marbles or basketballs.

What Avogadro had in mind is not really so mysterious if you consider that the particles in a gas are very far apart, with nothing but space in between. Thus a collection of relatively large particles does not require much more space than the same number of relatively small particles. On average, there would be large expanses of space between the particles in either case, as shown in **Figure 12.21.** This was Avogadro's great insight, today easily demonstrated by experiment. At STP, 1 mol (6.02×10^{23}) of particles of any gas, regardless of the size of the particles, occupies a volume of 22.4 L.

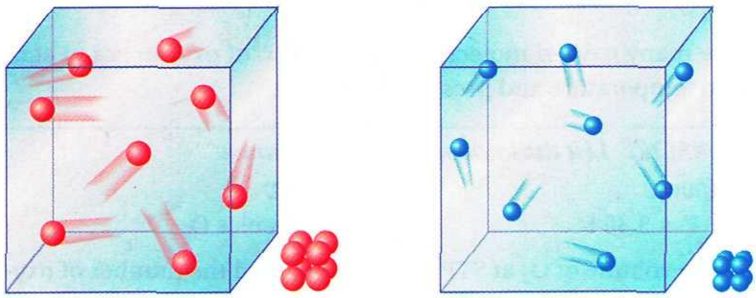


Figure 12.21

The volume of a container easily accommodates the same number of relatively large or small particles, as long as the particles are not tightly packed. There is a great deal of empty space compared with the volume occupied by the particles. When the particles are tightly packed, large particles take up more space than small particles.

You can also understand Avogadro's hypothesis by thinking about the modern explanation for gas pressure. Equal numbers of particles of different gases in equal volumes at the same temperature should exert the same pressure because the particles have the same average kinetic energy and are contained within equal volumes. Thus whenever you have equal volumes of gases at the same temperature and pressure, the volumes should contain equal numbers of particles.

Sample Problem 12-7

Determine the volume (in L) occupied by 0.202 mol of a gas at standard temperature and pressure (STP).

1. *ANALYZE* *List the known and the unknown.*

Known:
- $n = 0.202$ mol

Unknown:
- $V = ?$ L

The number of moles of gas at STP is given. Convert from moles to the volume (V).

$$\text{moles} \longrightarrow \text{volume}$$

The conversion factor for moles ⟶ volume at STP is 22.4 L/1 mol.

2. *CALCULATE* *Solve for the unknown.*

Multiplying the known value by the conversion factor yields:

$$V = 0.202\ \cancel{\text{mol}} \times \frac{22.4\ \text{L}}{1\ \cancel{\text{mol}}} = 4.52\ \text{L}$$

3. *EVALUATE* *Does the result make sense?*

Because 1 mol of the gas occupies 22.4 L at STP, 0.202 mol of the gas should occupy about one-fifth of that volume, or about 4.5 L.

Practice Problems

31. What is the volume occupied by 0.250 mol of a gas at STP?

32. What volume does 0.742 mol of argon gas occupy at STP?

Sample Problem 12-8

How many oxygen molecules are in 3.36 L of oxygen gas at standard temperature and pressure (STP)?

1. *ANALYZE* *List the known and the unknown.*

Known:
- $V = 3.36$ L

Unknown:
- ? molecules O_2

The volume of O_2 at STP is known. To find the number of oxygen molecules, make the following conversion.

$$\text{volume} \longrightarrow \text{moles} \longrightarrow \text{molecules}$$

The conversion factors are 1 mol O_2/22.4 L O_2 and 6.02×10^{23} molecules O_2/1 mol O_2, respectively.

Practice Problems

33. How many nitrogen molecules are in 5.12 L of the gas at STP?

Sample Problem 12-8 (cont.)

2. CALCULATE ***Solve for the unknown.***
Multiplying the known value by the conversion factors yields:

$$3.36\,\cancel{L\,O_2} \times \frac{1\,\cancel{mol\,O_2}}{22.4\,\cancel{L\,O_2}} \times \frac{6.02 \times 10^{23}\text{ molecules }O_2}{1\,\cancel{mol\,O_2}}$$
$$= 9.03 \times 10^{22}\text{ molecules }O_2$$

3. EVALUATE ***Does the result make sense?***
The answer is reasonable: 3.36 L O_2 should contain about one-seventh mole of O_2 molecules. One-seventh of approximately 6×10^{23} molecules is about 9×10^{22} molecules.

Practice Problems (cont.)

34. What volume is occupied by 4.02×10^{22} molecules of helium gas at STP?

Chem ASAP!
Problem-Solving 34
Solve Problem 34 with the help of an interactive guided tutorial.

Sample Problem 12-9

Determine the volume (in L) occupied by 14.0 g of nitrogen gas at STP.

1. ANALYZE ***List the known and the unknown.***

Known:
- mass = 14.0 g $N_2(g)$

Unknown:
- $V = ?$ L

The mass of nitrogen is known. Convert the known mass to volume.

mass ⟶ moles ⟶ volume

The conversion factors are 1 mol N_2/molar mass N_2 and 22.4 L N_2/1 mol N_2, respectively. The conversion factors require the value of the molar mass of N_2.

2. CALCULATE ***Solve for the unknown.***
First find the molar mass of N_2.

$$\text{molar mass } N_2 = 2\,\cancel{mol\,N} \times \frac{14.0\text{ g N}}{1\,\cancel{mol\,N}}$$
$$= 28.0\text{ g }N_2$$

Substitute the molar mass into the conversion factor, then solve for the volume of nitrogen gas.

$$14.0\,\cancel{g\,N_2} \times \frac{1\,\cancel{mol\,N_2}}{28.0\,\cancel{g\,N_2}} \times \frac{22.4\text{ L }N_2}{1\,\cancel{mol\,N_2}}$$
$$= 11.2\text{ L }N_2$$

3. EVALUATE ***Does the result make sense?***
The mass of nitrogen corresponds to one-half mole of N_2. One-half mole of gas at STP should occupy half of 22.4 L, or 11.2 L.

Practice Problems

35. What is the volume of a container that holds 8.80 g of carbon dioxide at STP?

36. A container holds 6.92 g of hydrogen gas at STP. What is the volume of the container?

Dalton's Law

The gases listed in **Table 12.1** make up a mixture called air. The particles in a gas mixture at the same temperature have the same average kinetic energy. Gas pressure depends only on the number of gas particles in a given volume and on their average kinetic energy—the kind of particle is unimportant. Each particle makes the same contribution to the pressure. Thus if you know the pressure exerted by each gas in a mixture, you can add the individual pressures to get the total gas pressure.

Table 12.1

Composition of Dry Air		
Component	**Volume (%)**	**Partial pressure (kPa)**
Nitrogen	78.08	79.11
Oxygen	20.95	21.22
Carbon dioxide	0.04	0.04
Argon and others	0.93	0.95
	100.00	101.32

Chem ASAP!

Animation 13
Observe the behavior of a mixture of nonreacting gases.

The contribution each gas in a mixture makes to the total pressure is called the **partial pressure** exerted by that gas. In a mixture of gases, the total pressure is the sum of the partial pressures of the gases.

$$P_{total} = P_1 + P_2 + P_3 + \ldots$$

This equation is one mathematical form of **Dalton's law of partial pressures:** At constant volume and temperature, the total pressure exerted by a mixture of gases is equal to the sum of the partial pressures of the component gases. The individual gases in containers a, b, and c of **Figure 12.22** are combined in container T. What partial pressure does each individual gas contribute to the total pressure of the mixture?

The fractional contribution to pressure exerted by each gas in a mixture does not change as the temperature, pressure, or volume changes. This fact has important implications for aviators and mountain climbers. For example, on top of Mount Everest, the total atmospheric pressure is reduced to 33.73 kPa (about one-third of its value at sea level). The partial pressure of oxygen is reduced by the same factor, to only 7.06 kPa (one-third the partial pressure of oxygen at sea level). This reduced oxygen

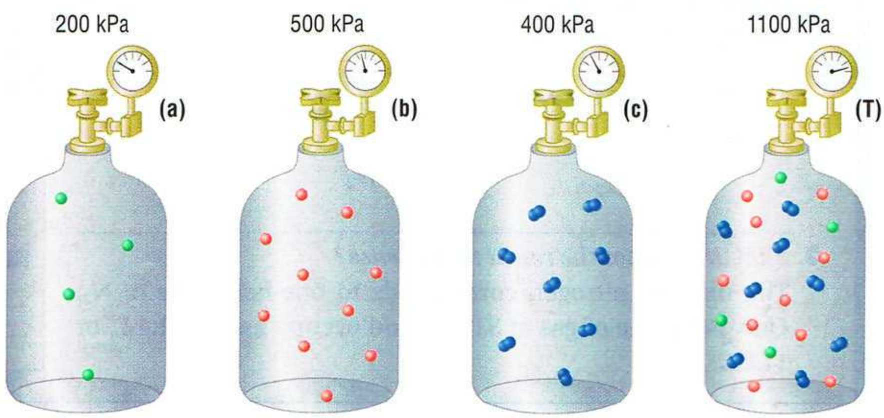


Figure 12.22
The sum of the pressures exerted by the gas in each of the three containers on the left is the same as the total pressure exerted by a mixture of the gases in the same volume, assuming the temperature stays the same. Dalton's law of partial pressures holds true because each gas exerts its own pressure independent of the pressure exerted by the other gases.

pressure is insufficient for respiration because humans need an oxygen partial pressure of at least 10.67 kPa; some individuals need a higher partial pressure. **Figure 12.23** shows some of the steps jet pilots and mountaineers take to counteract high-altitude conditions.

Figure 12.23
High-altitude pilots and mountaineers must have supplemental oxygen supplies available when at high altitudes.

Sample Problem 12-10

Air contains oxygen, nitrogen, carbon dioxide, and trace amounts of other gases. What is the partial pressure of oxygen (P_{O_2}) at 101.30 kPa of total pressure if the partial pressures of nitrogen, carbon dioxide, and other gases are 79.10 kPa, 0.040 kPa, and 0.94 kPa, respectively.

1. **ANALYZE** ***List the knowns and the unknown.***

Knowns:
- P_{N_2} = 79.10 kPa
- P_{CO_2} = 0.040 kPa
- P_{others} = 0.94 kPa
- P_{total} = 101.30 kPa

Unknown:
- P_{O_2} = ? kPa

Use the known values and Dalton's law of partial pressures ($P_{total} = P_{O_2} + P_{N_2} + P_{CO_2} + P_{others}$) to calculate the unknown value (P_{O_2}).

2. **CALCULATE** ***Solve for the unknown.***

Rearrange the expression for Dalton's law to isolate P_{O_2}. Substitute the values for the partial pressures and solve the equation.

$$P_{O_2} = P_{total} - (P_{N_2} + P_{CO_2} + P_{others})$$
$$= 101.30 \text{ kPa} - (79.10 \text{ kPa} + 0.040 \text{ kPa} + 0.94 \text{ kPa})$$
$$= 21.22 \text{ kPa}$$

3. **EVALUATE** ***Does the result make sense?***

The partial pressure of oxygen must be smaller than that of nitrogen because P_{total} is only 101.30 kPa. The other partial pressures are small, so an answer of 21.22 kPa seems reasonable.

Practice Problems

37. Determine the total pressure of a gas mixture that contains oxygen, nitrogen, and helium if the partial pressures of the gases are as follows: P_{O_2} = 20.0 kPa, P_{N_2} = 46.7 kPa, and P_{He} = 26.7 kPa.

38. A gas mixture containing oxygen, nitrogen, and carbon dioxide has a total pressure of 32.9 kPa. If P_{O_2} = 6.6 kPa and P_{N_2} = 23.0 kPa, what is P_{CO_2}?

Chem ASAP!

Problem-Solving 38
Solve Problem 38 with the help of an interactive guided tutorial.

Figure 12.24
White vapors of ammonium chloride form as the vapors from aqueous ammonia (NH_3)(aq) and hydrochloric acid (HCl)(aq) diffuse into the air. Aqueous ammonia used to be called ammonium hydroxide, a name seldom used today. What are the names and formulas of the molecules in the vapors? Which molecule diffuses faster? Why?

Graham's Law

When you open a perfume bottle inside a room, the perfume molecules eventually spread throughout the room, and you can smell them everywhere. **Diffusion** is the tendency of molecules to move toward areas of lower concentration until the concentration is uniform throughout. **Figure 12.25** illustrates the diffusion process for bromine vapor. The concentrated bromine vapor tends to move outside the graduated cylinder toward an area where the bromine vapor concentration is less.

Much of the early work on diffusion was done in the 1840s by the Scottish chemist Thomas Graham (1805–1869). Graham measured the rates of **effusion,** the process in which a gas escapes through a tiny hole in its container. Graham noticed that gases of lower molar mass effuse faster than gases of higher molar mass. From his observations, he proposed **Graham's law of effusion:** The rate of effusion of a gas is inversely proportional to the square root of the gas's molar mass. Subsequently, this relationship was also shown to be true for the diffusion of gases. Thus the rate of diffusion of a gas is also inversely proportional to the square root of its molar mass.

To understand Graham's law, examine the relationship of the mass and speed of a moving body to the kinetic energy the body transfers when it strikes a stationary object (assuming perfectly elastic collisions). The mathematical expression that relates the mass (m) and the speed or velocity (v) of a body to its kinetic energy (KE) is $KE = \frac{1}{2}mv^2$. Suppose a small ball with a mass of 2 g traveling at 5 m/s has just enough kinetic energy to shatter a pane of glass. A ball bearing with a mass of only 1 g would need to travel faster (slightly more than 7 m/s) to have the same kinetic energy as the ball and to be able to shatter the same pane of glass.

Chem ASAP!

Animation 14

Examine the processes of gas effusion and diffusion.

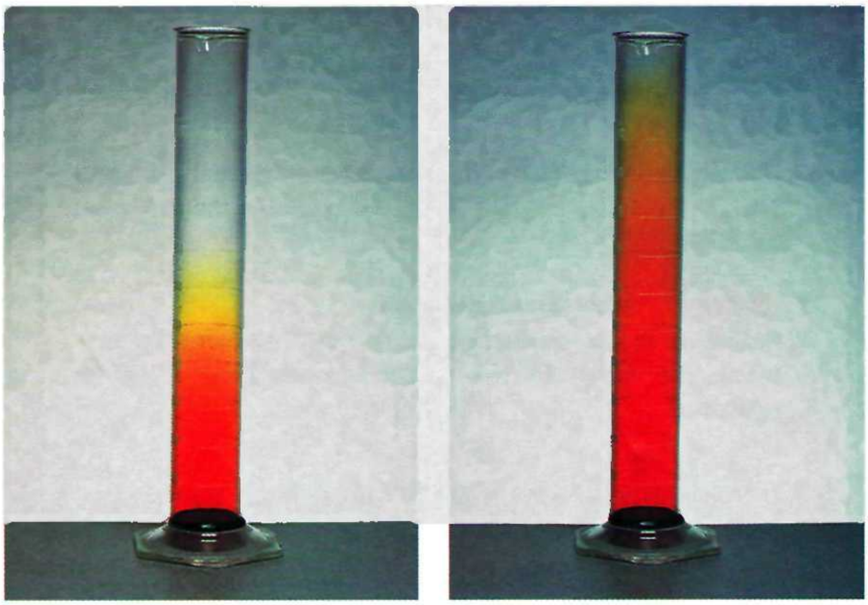


Figure 12.25
The diffusion of one substance through another is a relatively slow process. Here, bromine vapor is diffusing upward through the air in a graduated cylinder. After several hours, bromine vapors will mingle with air outside the cylinder.

There is an important principle here. If two bodies of different masses have the same kinetic energy, the lighter body must move faster. You already know that the particles of two different gases at the same temperature have the same average kinetic energy. Thus a gas particle of low mass should move faster than a gas particle of high mass if the gases are at the same temperature. The gas of lower molar mass should therefore diffuse and effuse faster.

It is easy to show that the above phenomenon is true by comparing two balloons: one filled with helium and the other filled with air. There are pores in a balloon that, although tiny, are still large enough for both helium atoms and molecules in air to pass through freely. Balloons filled with air stay inflated longer, because the main components of air—oxygen molecules and nitrogen molecules—are more massive than helium atoms. The air particles move more slowly; therefore, they diffuse and effuse more slowly than helium atoms. The fast-moving helium atoms, with molar masses of only 4 g, rapidly effuse through the pores in the balloon. The rate of effusion or diffusion is related only to the particle's speed. In a mathematical form, Graham's law can be written as follows for two gases, A and B.

$$\frac{\text{Rate}_A}{\text{Rate}_B} = \frac{\sqrt{\text{molar mass}_B}}{\sqrt{\text{molar mass}_A}}$$

In other words, the rates of effusion of two gases are inversely proportional to the square roots of their molar masses. Now compare the rates of effusion of the air component nitrogen (molar mass = 28.0 g) and helium (molar mass = 4.0 g):

$$\frac{\text{Rate}_{He}}{\text{Rate}_{N_2}} = \frac{\sqrt{28.0\text{ g}}}{\sqrt{4.0\text{ g}}} = \frac{5.3\cancel{\text{g}}}{2.0\cancel{\text{g}}} \approx 2.7$$

You can see that helium effuses and diffuses nearly three times faster than nitrogen at the same temperature.

portfolio project

To estimate the amount of CO_2 in a soft drink, obtain an unopened bottle and a large spherical balloon. Open the bottle and quickly place the balloon over its neck. Fasten the balloon tightly and shake the bottle gently for at least 5 minutes. Use the volume of a sphere to estimate the volume of CO_2. Use the gas laws to find the number of moles of CO_2. Compare different brands. Identify possible sources of error in this experiment.

section review 12.5

39. In your own words, briefly state Avogadro's hypothesis, Dalton's law, and Graham's law.

40. How are moles, masses, or volumes of gases calculated from one another for a gas at STP?

41. Calculate the number of liters occupied at STP.

a. 1.7 mol $H_2(g)$

b. 1.8×10^{-2} mol $N_2(g)$

c. 2.5×10^{2} mol $O_2(g)$

42. How is the partial pressure of a gas in a mixture calculated? How is the rate of effusion of a gas calculated?

43. What is the significance of the volume 22.4 L?

44. At the same temperature, the rates of diffusion of carbon monoxide and nitrogen gas are virtually identical. Explain.

Chem ASAP! **Assessment 12.5** Check your understanding of the important ideas and concepts in Section 12.5.